ECOM 111

# The continuity equation and minority carrier diffusion equation

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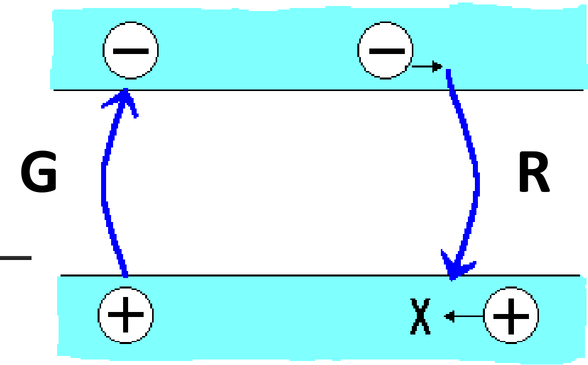
# Generation and Recombination




## Generation

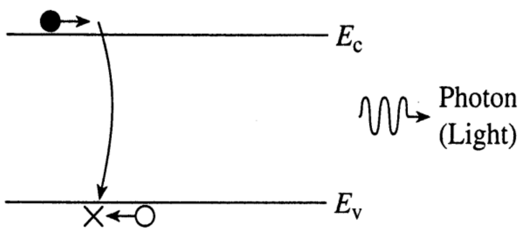
- Process whereby electrons and holes (carriers) are created.
- Photogeneration - light energy
- Direct thermal generation - heat energy

## Recombination

- Process whereby electrons and holes are annihilated or destroyed.
- Direct thermal recombination

# Recombination Mechanisms

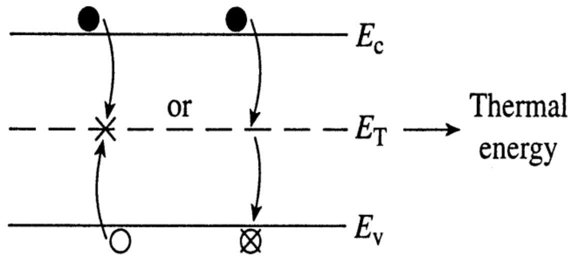
## Band-to-band:



(a) Band-to-band recombination

- e- and h+ annihilate each other and produce a photon

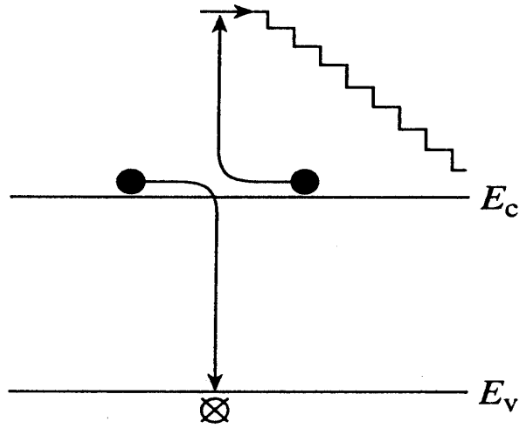
## R-G centers



(b) R–G center recombination

- 2-step process involving a lattice defect or impurity (the RG center)
- carrier is "trapped" by R-G center; another carrier is attracted by trapped one
- also called "indirect recombination"

## Auger

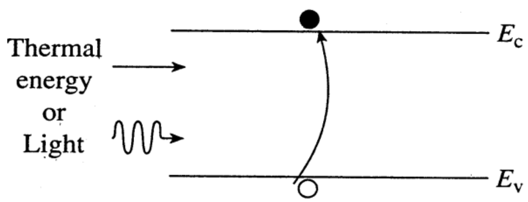


(c) Auger recombination

- band-to-band recombination simultaneous with collision between like carriers
- energy gained during collision by surviving carrier lost in small steps through collisions with the lattice

# Generation Mechanisms

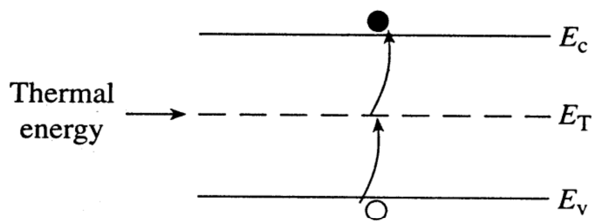
## Band-to-band




(d) Band-to-band generation

- e- and h+ produced by thermal energy or by a photon "photogeneration"

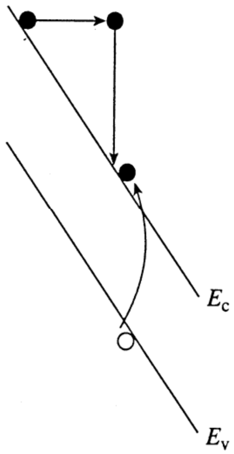
## R-G centers




(e) R–G center generation

- thermally assisted process involving an R-G center acting as an intermediary

## Impact ionization




(f) Carrier generation via impact ionization

- band-to-band generation caused by collision between carrier and lattice
- occurs at high E-fields

# Introduction

$$\frac{dn}{dt} = G - R$$

$G$ : Generation rate

$R$ : Recombination rate

$$R \propto n$$

$$R \propto p$$

$$R = rnp$$

$r$ : Recombination coefficient

$$\boxed{\frac{dn}{dt} = G - rnp}$$

For **thermal equilibrium** , the carrier concentration doesn't change $\frac{dn}{dt} = 0$

$$0 = G - rnp \Rightarrow G_o = rn_op_o \Rightarrow G_o = r\,n_i^2$$

$G_o$ is the thermal generation rate

$n_o\,, p_o$ Electrons and holes thermal equilibrium concentration

# (dn/dt) → Recombination

Consider a semiconductor that is **exposed to the light for a long time** and **suddenly** the **light source is removed**

$t < 0$ : SC is exposed to light

$t = 0$ : Light is switched off

For $t > 0$ "After the light is witched off"

$$n = n_o + \Delta n$$

$$p = p_o + \Delta p$$

$$\frac{dn}{dt} = G - R$$

$$\frac{dn}{dt} = G - rnp$$

$$\frac{dn}{dt} = G_o - r(n_o + \Delta n)(p_o + \Delta p)$$

$$\frac{dn}{dt} = G_o - rn_o p_o - rn_o \Delta p - rp_o \Delta n - r\Delta n \Delta p$$

$$\frac{dn}{dt} = -rn_o\Delta p - rp_o\Delta n$$

Consider we are studying a **p-type** material

$$n_o \ll p_o$$

$$\frac{dn}{dt} = -rp_o\Delta n$$

$$\frac{dn}{dt} = \frac{d(n_o + \Delta n)}{dt} = \frac{d\Delta n}{dt}$$

$$\frac{d\Delta n}{dt} = -rp_o\Delta n$$

Let $\tau_e = \dfrac{1}{rp_o}$

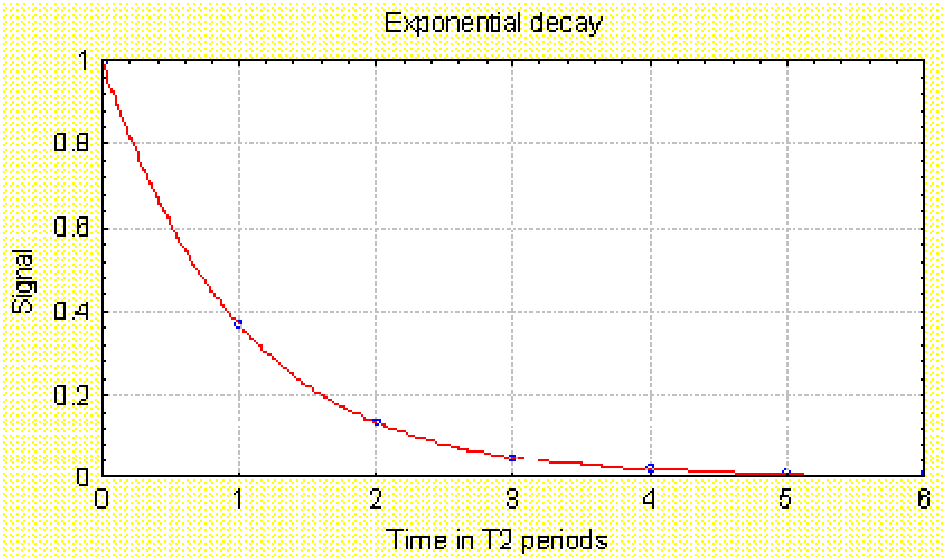
$$\boxed{\frac{d\Delta n}{dt} = -\frac{\Delta n}{\tau_e}} \Rightarrow \frac{d\Delta n}{\Delta n} = -\frac{dt}{\tau_e}$$

$$\ln \Delta n = \frac{-t}{\tau_e} + C$$

$$\Delta n = e^{\frac{-t}{\tau_e}} \; e^{C}$$

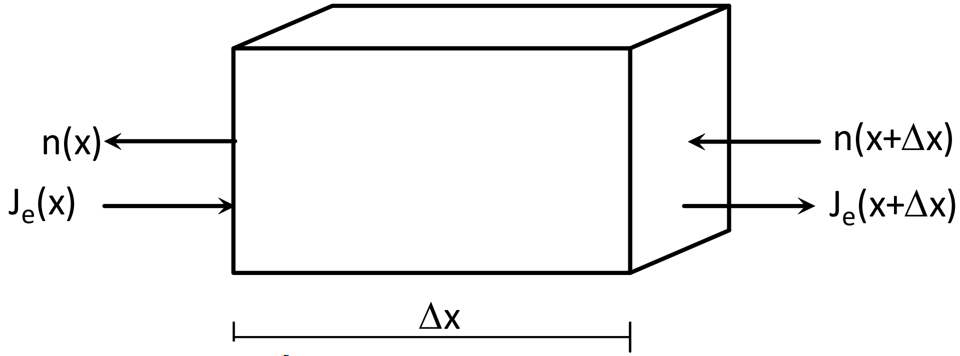
Let $e^{C} = \Delta n_o$

$$\boxed{\Delta n(t) = \Delta n_o \, e^{\frac{-t}{\tau_e}}}$$



# $(dn/dt) \rightarrow$ Current

Now, consider **a current** passes through the semi conductor




The relationship between $J$ and $n$

$$J = qnv$$

$$J_e(x) = n(x)qv$$

$$J_e(x+\Delta x) = n(x+\Delta x)qv$$

$$\Delta n = n(x+\Delta x) - n(x)$$

$$\Delta n = \frac{1}{qv}[J_e(x+\Delta x) - J_e(x)]$$

$$\Delta n = \frac{1}{q\frac{\Delta x}{\Delta t}}[J_e(x+\Delta x) - J_e(x)]$$

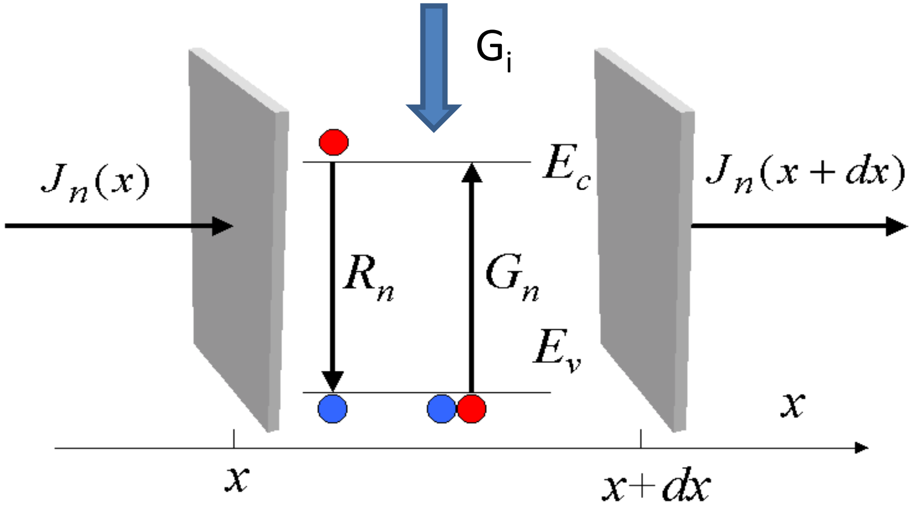
$$\frac{\Delta n}{\Delta t} = \frac{1}{q}\ \frac{[J_e(x+\Delta x) - J_e(x)]}{\Delta x}$$

$$\frac{\Delta n}{\Delta t} = \frac{1}{q}\ \frac{\Delta J}{\Delta x}$$

$$\boxed{\frac{dn}{dt} = \frac{1}{q}\ \frac{dJ}{dx}}$$

$$\frac{dn}{dt} = \left(\frac{1}{q}\frac{d}{dx}\left(q\mu_n nE + qD_n\frac{dn}{dx}\right)\right)$$

# Continuity equations




$$\frac{\partial n}{\partial t} = \frac{\partial n}{dt}\bigg|_{\text{current}} + \frac{\partial n}{\partial t}\bigg|_{\text{recombination}} + \frac{\partial n}{dt}\bigg|_{\text{Light}}$$

$$\boxed{\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J}{\partial x} - \frac{\Delta n}{\tau_e} + G_i}$$

Conitnuity Equation for electrons

$$\boxed{\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J}{\partial x} - \frac{\Delta p}{\tau_h} + G_i}$$

Conitnuity Equation for holes

# Simplification on Continuity equations

- One dimensional case
- We will consider minority carries
- No applied electric field ( E=0) (i.e. no drift current)
- Low Level Injection

The continuity equation becomes **the minority carriers diffusion equation**

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial}{\partial x}\left[qD_e\frac{\partial n}{\partial x}\right] - \frac{\Delta n}{\tau_e} + G_i$$

$$\frac{\partial n}{dt} = D_e\frac{\partial^2 n}{\partial x^2} - \frac{\Delta n}{\tau_e} + G_i$$

$$\boxed{\frac{\partial \Delta n}{\partial t} = D_e\frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_e} + G_i}$$

**Similarly** $\Rightarrow$

$$\boxed{\frac{\partial \Delta p}{\partial t} = D_h\frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta n}{\tau_h} + G_i}$$

# Special cases and Solutions

$$\boxed{\frac{\partial \Delta n}{\partial t} = D_e \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_e} + G_i}$$

$$\boxed{\frac{\partial \Delta p}{\partial t} = D_h \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta n}{\tau_h} + G_i}$$

$\boxed{1}$ Steady state , no light

$$\frac{\partial}{\partial t} = 0 \quad , \quad G_i = 0$$

$$D_e \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_e} = 0$$

$$\frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{D_e \tau_e} = 0$$

$$\frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{{L_e}^2} = 0$$

The solution will be on the form

$$\boxed{\Delta n(x) = C_1 e^{-\frac{x}{L_e}} + C_2 e^{+\frac{x}{L_e}}}$$

$$\boxed{\frac{\partial \Delta n}{\partial t} = D_e \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_e} + G_i}$$

$$\boxed{\frac{\partial \Delta p}{\partial t} = D_h \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta n}{\tau_h} + G_i}$$

## 2 No concentration gradient , no light

$$\frac{\partial}{\partial x} = 0 \quad , \quad G_i = 0$$

$$\frac{\partial \Delta n}{\partial t} = -\frac{\Delta n}{\tau_e}$$

$$\frac{\partial \Delta n}{\Delta n} = -\frac{\partial t}{\tau_e}$$

$$\ln \Delta n = \frac{-t}{\tau_e} + C$$

$$\Delta n = e^{\frac{-t}{\tau_e}} \ e^{C}$$

$$\boxed{\Delta n(t) = \Delta n_o \, e^{\frac{-t}{\tau_e}}}$$

$$\boxed{\frac{\partial \Delta n}{\partial t} = D_e \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_e} + G_i}$$

$$\boxed{\frac{\partial \Delta p}{\partial t} = D_h \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta n}{\tau_h} + G_i}$$

## 3 Steady state , no concentration gradient

$$\frac{\partial}{\partial t} = 0 \quad , \quad \frac{\partial}{\partial x} = 0$$

$$0 = -\frac{\Delta n}{\tau_e} + G_i$$

$$\boxed{\Delta n = G_i \tau_e}$$

$$\boxed{\frac{\partial \Delta n}{\partial t} = D_e \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_e} + G_i}$$

$$\boxed{\frac{\partial \Delta p}{\partial t} = D_h \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta n}{\tau_h} + G_i}$$

## 4 Steady state , no recombination , no light

$$\frac{\partial}{\partial t} = 0 \quad , \quad \tau_e = \infty \quad , \quad G_i = 0$$

$$D_e \frac{\partial^2 \Delta n}{\partial x^2} = 0$$

$$\frac{\partial^2 \Delta n}{\partial x^2} = 0$$

$$\frac{\partial \Delta n}{\partial x} = C_1$$

$$\partial \Delta n = C_1 \, \partial x$$

$$\boxed{\Delta n(x) = C_1 \, x + C_2}$$

$$\boxed{\frac{\partial \Delta n}{\partial t} = D_e \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_e} + G_i}$$

$$\boxed{\frac{\partial \Delta p}{\partial t} = D_h \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta n}{\tau_h} + G_i}$$

## 5 Steady state , no recombination

$$\frac{\partial}{\partial t} = 0 \quad , \quad \tau_e = \infty$$

$$D_e \frac{\partial^2 \Delta n}{\partial x^2} = -G_i$$

$$\frac{\partial^2 \Delta n}{\partial x^2} = -\frac{G_i}{D_e}$$

$$\frac{\partial \Delta n}{\partial x} = -\frac{G_i}{D_e} x + C_1$$

$$\partial \Delta n = \left( -\frac{G_i}{D_e} x + C_1 \right) \partial x$$

$$\boxed{\Delta n(x) = -\frac{G_i}{2D_e} x^2 + C_1\, x + C_2}$$

$$\frac{\partial \Delta n}{\partial t} = D_e \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_e} + G_i$$

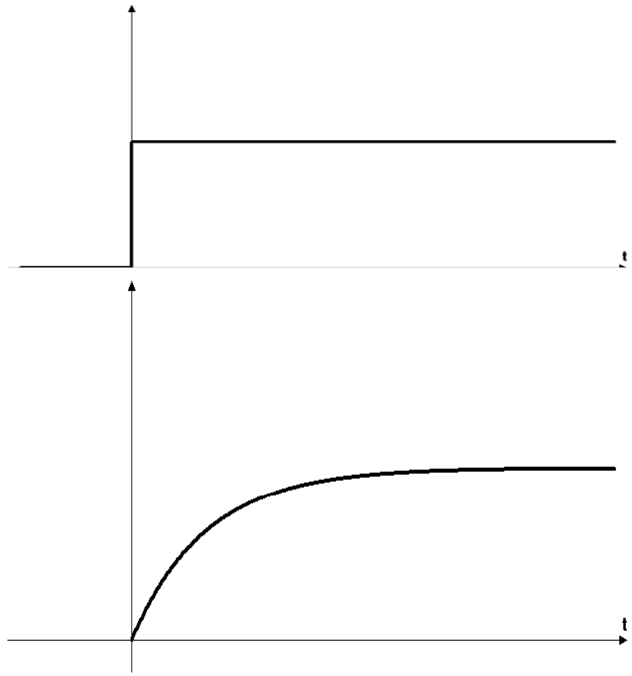
$$\frac{\partial \Delta p}{\partial t} = D_h \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta n}{\tau_h} + G_i$$

## 6 Transient (No concentration gradient)

When the light is just applied to the semiconductor

$$\frac{\partial}{\partial x} = 0$$

$$\frac{\partial \Delta n}{\partial t} = -\frac{\Delta n}{\tau_e} + G_i$$




The solution will be on the form

$$\boxed{\Delta n(t) = G_i \tau_e \left[1 - e^{\frac{-t}{\tau_e}}\right]}$$

This equation represents exponential rise

After $5\tau$ , $\Delta n(t) = G_i \tau_e$ "Steady-state value"

i.e. When the light illuminates the semiconductor sample for a very large time $\Delta n$ reaches its steady state value